SOLUTION Put your grapher in parametric mode. Then enter

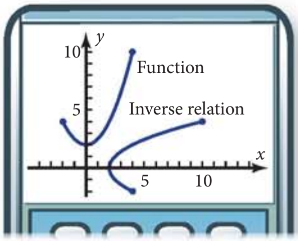

Figure 1-5h

$x_1(t) = t$

$y_1(t) = 0.5t^2 + 2$ Because $x = t$, this is equivalent to $y = 0.5x^2 + 2$.

$x_2(t) = 0.5t^2 + 2$

$y_2(t) = t$ For the inverse, interchange the equations for x and y.

Use a window with $-2 \le t \le 4$. Use a convenient t-step, such as 0.1. The result is shown in Figure 1-5h.

The range of the inverse relation is the same as the domain of the function, and vice versa. The range and the domain are interchanged.

Example 4 shows you how to demonstrate algebraically that $f^{-1}(f(x)) = x$ and $f\left(f^{-1}(x)\right) = x$.

EXAMPLE 4 Let $f(x) = 3x + 12$.

a. Find an equation for the inverse of f, and explain how that equation confirms that f is an invertible function.

b. Demonstrate that $f^{-1}(f(x)) = x$ and $f\left(f^{-1}(x)\right) = x$.

SOLUTION

a. Function: $y = 3x + 12$

Inverse: $x = 3y + 12 \Rightarrow y = \frac{1}{3}x - 4$

Because the equation for the inverse relation has the form $y = mx + b$, the inverse is a linear function. Because the inverse relation is a function, f is invertible, so the equation can be written

$$f^{-1}(x) = \frac{1}{3}x - 4$$

b. $f^{-1}(f(x)) = f^{-1}(3x + 12)$ Substitute $3x + 12$ for $f(x)$.

$= \frac{1}{3}(3x + 12) - 4$ Substitute $3x + 12$ as the input for function f^{-1}.

$= x + 4 - 4 = x$ Show that $f^{-1}(f(x))$ equals x.

Also, $f\left(f^{-1}(x)\right) = f\left(\frac{1}{3}x - 4\right)$ Show that $f\left(f^{-1}(x)\right)$ equals x.

$= 3\left(\frac{1}{3}x - 4\right) + 12$

$= x - 12 + 12 = x$

$\therefore f^{-1}(f(x)) = x$ and $f\left(f^{-1}(x)\right) = x$, Q.E.D.

Note: The three-dot mark $\therefore$ stands for "therefore." The letters Q.E.D. stand for the Latin words *quod erat demonstrandum*, meaning "which was to be demonstrated."

The box on the next page summarizes the information of this section regarding inverses of functions.